MAP554 – Networks: PC 3

28 September 2016

1. α-fair allocations

Consider a network constituted of L links $\ell = 1, \dots, L$ in series, of unit capacity, in which type 0 users use all links simultaneously, and for each $s = 1, \dots, L$, type s users use only link s.

1.1 Assume there are n_s users of type s for $s = 0, \dots, L$. Let parameter $\alpha > 0$ be fixed. Explain why the $(1, \alpha)$-fair allocation of capacities to all users gives the same capacity for any two users of the same type. Denoting by λ_s the $(1, \alpha)$-fair rate that each type s user receives, write the optimization programme that characterizes these rates.

1.2 Compute (using Lagrange multipliers) these rates λ_s.

1.3 Show that as $\alpha \to 1$ (respectively, $\alpha \to +\infty$), these rates converge to proportionally fair rates (respectively, max-min fair rates).

1.4 Show that the same convergence results as $\alpha \to 1$ or $+\infty$ hold more generally for an arbitrary network model, not just the linear one under consideration.

Hint: consider the optimal allocations $\lambda_s(\alpha)$ as functions of associated Lagrange multipliers $p_\ell(\alpha)$. In the case $\alpha \to \infty$, consider $q_\ell(\alpha) := [p_\ell(\alpha)]^{1/\alpha}$ and argue that these are bounded.

2. Another reverse engineering of TCP

We consider a network of TCP-controlled traffic sources $s\mathcal{S}$, each characterized by a round-trip time T_s, a congestion window $cwnd_s$, a sending rate $x_s = cwnd_s/T_s$. We assume its sent packets traverse linkes $\ell \in s$, and incur losses at a rate $x_s p_s$, where $p_s = \sum_{\ell \in s} p_\ell$ and $p_\ell = C'_\ell(\sum_{s' \ni \ell} x_{s'})$ is the loss probability at link ℓ, assumed to be a function C'_ℓ of that link's total load $\sum_{s' \ni \ell} x_{s'}$. One would then interpret C'_ℓ as the derivative of a cost function $C_\ell(\cdot)$.

2.1 Arguing that an increase by $1/cwnd_s$ of the congestion window $cwnd_s$ occurs at rate $x_s(1 - p_s)$, and a decrease by $cwnd_s/2$ occurs at rate $x_s p_s$, justify the ODE model for the evolution of x_s:

$$\frac{d}{dt} x_s = \frac{1}{T_s^2}(1 - p_s) - \frac{x_s^2}{2} p_s.$$

2.2 Explain why the above ODE corresponds to a primal algorithm optimizing total welfare $\sum_s U_s(x_s) - \sum_\ell C_\ell(\sum_{s' \ni \ell} x_{s'})$ for utility functions U_s to be determined.

3. Reverse engineering of fixed-window congestion control

Consider a collection of users s, each using an associated collection of network links $\ell \in s$. Let C_ℓ be the capacity of link ℓ. Let W_s be the *window* size associated with user s. We assume here the following fixed window control: each user s sends W_s packets without waiting for an acknowledgement. These packets traverse the links $\ell \in s$, and then induce an acknowledgement sent back to user s, who re-injects a new packet upon receipt of each acknowledgement.

Assume a fluid model, with for each user s an associated rate λ_s of ack receptions and packet injections. Assume a static regime, where for each $\ell \in s$, the rate of packet arrivals from user s is constant, equal to λ_s. Let B_ℓ be the amount of traffic queued at link ℓ; and let $B_{\ell,s}$ be the amount that is queued that was sent by user s.

3.1 Justify heuristically why in such a static model one should have the relations $C_\ell \geq \sum_{s \ni \ell} \lambda_s$ and $B_\ell[C_\ell \geq \sum_{s \ni \ell} \lambda_s] = 0$.

3.2 Explain why, assuming packets are handled at each link in a First-In-First-Out (FIFO) manner, in such a fluid regime one should have for each $\ell \in s$,

$$B_{\ell,s} = B_s \frac{\lambda_s}{\sum_{s' \ni \ell} \lambda_{s'}}.$$

3.3 Assume that for each s, sent and not yet acknowledged packets are all in one of the buffers of some link $\ell \in s$, so that $W_s = \sum_{\ell \in s} B_{\ell,s}$.

Characterize rates λ_s satisfying all the above conditions as a weighted proportionally fair allocation, with weights to be determined.

3.4 In contrast with the previous question, assume now that for each s, letting T_s denote its round-trip time parameter, there are $\lambda_s T_s$ packets not yet acknowledged and not buffered at any link $\ell \in s$, so that $W_s = \lambda_s T_s + \sum_{\ell \in s} B_{\ell,s}$. Interpret the rates λ_s as solutions of a modified utility maximization problem.