Benchmarking the (1+1)-CMA-ES on the BBOB-2009 Noisy Testbed

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ABSTRACT

We benchmark an independent-restart-(1+1)-CMA-ES on the BBOB-2009 noisy testbed. The (1+1)-CMA-ES is an adaptive stochastic algorithm for the optimization of objective functions defined on a continuous search space in a black-box scenario. The maximum number of function evaluations used here equals 10^4 times the dimension of the search space. The algorithm could only solve 4 functions with moderate noise in 5-D and 2 functions in 20-D.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation, CMA-ES

1. INTRODUCTION

The (1+1)-CMA-ES is an adaptive stochastic search algorithm combining the simple (1+1) selection scheme and the famous covariance matrix adaptation (CMA) mechanism [6]. This paper complements [1] where an independent-restart implementation of the (1+1)-CMA-ES is benchmarked on the BBOB-2009 noise-free testbed. Indeed we test exactly the same algorithm, using the same settings on the BBOB-2009 noisy testbed. For the description of the algorithm and the settings we refer to [1].

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2. RESULTS AND DISCUSSION

Results from experiments according to [4] on the benchmark functions given in [2, 5] are presented in Figures 1 and 2 and in Tables 1 and 2.

We observe that globally the algorithm performs poorly. In 5-D, only f_{101}, f_{102}, f_{103}, f_{104} are solved and in 20-D only f_{101} and f_{102} are solved. The functions solved belong to the class of functions with moderate noise. In comparison with the BI-population CMA-ES in [3], a restart algorithm using the $(\mu/\mu_{\mathrm{w}}, \lambda)$-CMA, the overall performance is poor. The (1+1) selection is an inferior choice for noisy optimization, because of the elitist selection and the lack of population. However, reevaluation of the parental solution and more allowed function evaluations might still leave some room for improvement.

Acknowledgments

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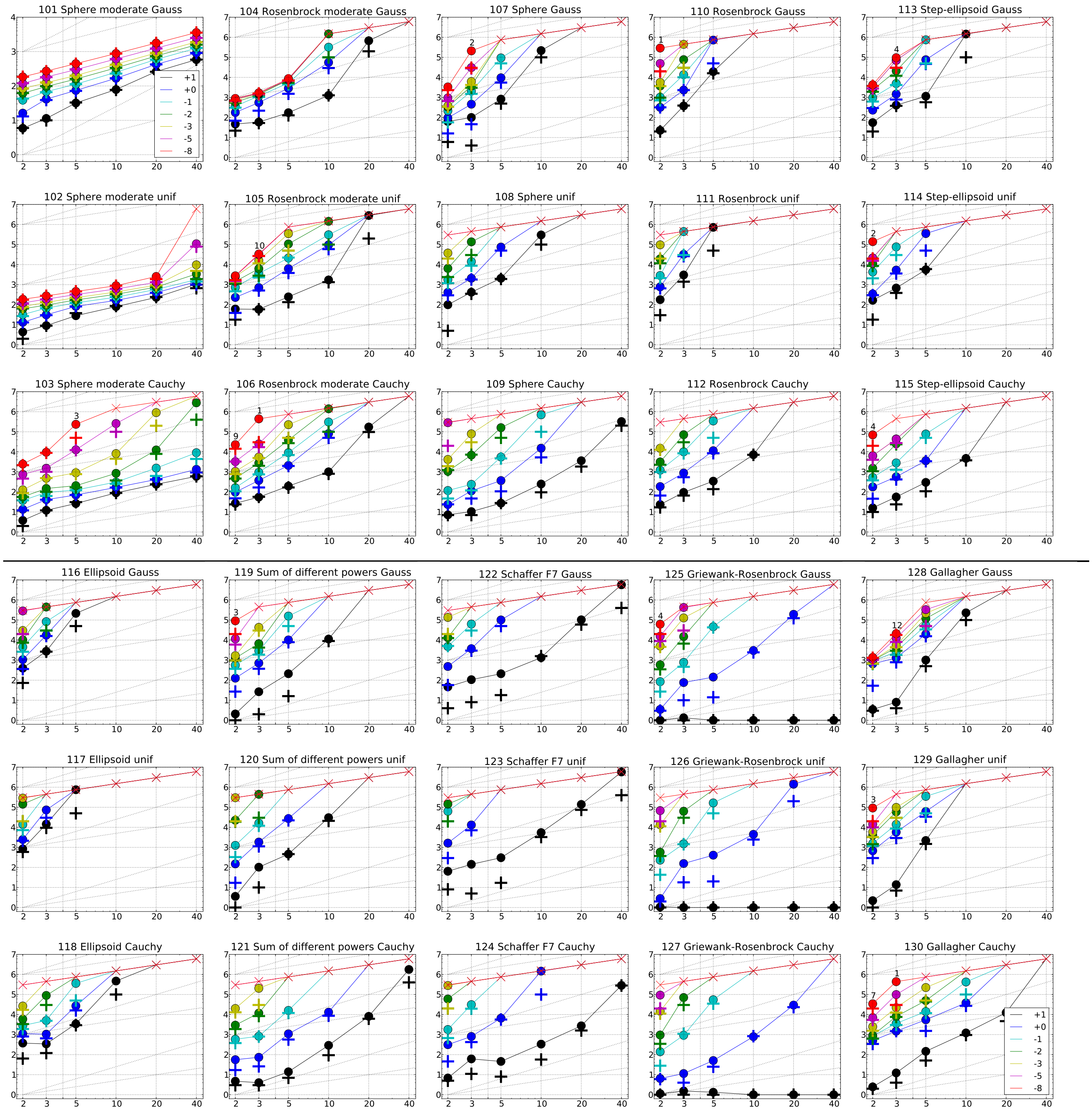

Figure 1: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\rm opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and $f_{\rm opt}$ denotes the optimal function value. Crosses (×) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

Δf	**f101 in 5-D**, N=15, mFE=499					**f101 in 20-D**, N=15, mFE=2020				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.2e1	2.7e1	3.7e1	3.2e1	15	2.6e2	2.5e2	2.7e2	2.6e2
1	15	7.4e1	6.7e1	8.1e1	7.4e1	15	4.2e2	4.1e2	4.3e2	4.2e2
1e−1	15	1.1e2	1.1e2	1.2e2	1.1e2	15	5.9e2	5.8e2	6.1e2	5.9e2
1e−3	15	2.0e2	1.9e2	2.1e2	2.0e2	15	9.1e2	9.0e2	9.3e2	9.1e2
1e−5	15	3.0e2	2.9e2	3.1e2	3.0e2	15	1.3e3	1.2e3	1.3e3	1.3e3
1e−8	15	4.5e2	4.4e2	4.6e2	4.5e2	15	1.8e3	1.7e3	1.8e3	1.8e3

Δf	**f103 in 5-D**, N=15, mFE=50001					**f103 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.6e1	2.1e1	3.1e1	2.6e1	15	2.4e2	2.3e2	2.5e2	2.4e2
1	15	7.0e1	6.5e1	7.5e1	7.0e1	15	4.2e2	4.1e2	4.3e2	4.2e2
1e−1	15	1.2e2	1.2e2	1.3e2	1.2e2	15	1.5e3	1.1e3	2.0e3	1.5e3
1e−3	15	9.0e2	6.7e2	1.1e3	9.0e2	3	9.0e5	5.2e5	2.8e6	1.5e5
1e−5	15	1.3e4	1.0e4	1.5e4	1.3e4	0	*16e−4*	*92e−5*	*28e−4*	7.9e4
1e−8	3	2.4e5	1.4e5	7.2e5	4.4e4	.	.	.	.	.

Δf	**f105 in 5-D**, N=15, mFE=50001					**f105 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.5e2	1.7e2	3.2e2	2.5e2	1	2.8e6	1.3e6	>3e6	2.0e5
1	15	6.3e3	3.8e3	9.2e3	6.3e3	0	*17e+0*	*10e+0*	*18e+0*	1.3e5
1e−1	15	2.1e4	1.8e4	2.5e4	2.1e4	.	.	.	.	.
1e−3	2	3.5e5	1.8e5	>7e5	5.0e4	.	.	.	.	.
1e−5	0	*16e−3*	*37e−6*	*83e−3*	2.5e4	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f107 in 5-D**, N=15, mFE=50001					**f107 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	8.3e2	4.1e2	1.3e3	8.3e2	0	*75e+0*	*48e+0*	*11e+1*	6.3e4
1	15	9.5e3	6.3e3	1.3e4	9.5e3	.	.	.	.	.
1e−1	6	9.4e4	6.1e4	1.6e5	3.5e4	.	.	.	.	.
1e−3	0	*19e−2*	*20e−3*	*43e−2*	2.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f109 in 5-D**, N=15, mFE=50001					**f109 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.7e1	2.2e1	3.3e1	2.7e1	15	3.6e3	2.4e3	5.0e3	3.6e3
1	15	3.7e2	2.3e2	5.2e2	3.7e2	0	*16e−1*	*12e−1*	*24e−1*	8.9e4
1e−1	15	5.5e3	4.0e3	7.2e3	5.5e3	.	.	.	.	.
1e−3	0	*20e−3*	*77e−4*	*66e−3*	2.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f111 in 5-D**, N=15, mFE=50001					**f111 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	1	7.2e5	3.5e5	>7e5	2.0e4	0	*44e+3*	*26e+3*	*73e+3*	1.3e5
1	0	*44e+0*	*12e+0*	*96e+0*	2.0e4	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f113 in 5-D**, N=15, mFE=50001					**f113 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.1e3	6.0e2	1.8e3	1.1e3	0	*22e+1*	*13e+1*	*28e+1*	1.0e5
1	8	7.6e4	5.7e4	1.1e5	4.2e4	.	.	.	.	.
1e−1	1	7.5e5	3.7e5	>7e5	5.0e4	.	.	.	.	.
1e−3	0	*96e−2*	*12e−2*	*17e−1*	3.2e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f115 in 5-D**, N=15, mFE=50001					**f115 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.0e2	2.0e2	4.1e2	3.0e2	0	*21e+0*	*12e+0*	*29e+0*	1.1e5
1	15	3.6e3	2.6e3	4.6e3	3.6e3	.	.	.	.	.
1e−1	7	7.8e4	5.4e4	1.2e5	3.6e4	.	.	.	.	.
1e−3	0	*11e−2*	*23e−3*	*37e−2*	2.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f117 in 5-D**, N=15, mFE=50001					**f117 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	1	7.5e5	3.7e5	>7e5	5.0e4	0	*18e+3*	*14e+3*	*25e+3*	7.1e4
1	0	*77e+0*	*21e+0*	*12e+1*	3.2e4	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f119 in 5-D**, N=15, mFE=50001					**f119 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.1e2	1.1e2	3.4e2	2.1e2	0	*17e+0*	*14e+0*	*21e+0*	8.9e4
1	15	1.0e4	6.9e3	1.4e4	1.0e4	.	.	.	.	.
1e−1	4	1.6e5	1.0e5	3.2e5	5.0e4	.	.	.	.	.
1e−3	0	*20e−2*	*69e−3*	*47e−2*	1.6e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f102 in 5-D**, N=15, mFE=492					**f102 in 20-D**, N=15, mFE=7206				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.8e1	2.3e1	3.3e1	2.8e1	15	2.3e2	2.2e2	2.4e2	2.3e2
1	15	8.2e1	7.8e1	8.7e1	8.2e1	15	4.0e2	3.9e2	4.1e2	4.0e2
1e−1	15	1.2e2	1.2e2	1.3e2	1.2e2	15	5.8e2	5.6e2	6.0e2	5.8e2
1e−3	15	2.2e2	2.2e2	2.3e2	2.2e2	15	9.4e2	9.3e2	9.6e2	9.4e2
1e−5	15	3.2e2	3.1e2	3.3e2	3.2e2	15	1.3e3	1.3e3	1.3e3	1.3e3
1e−8	15	4.6e2	4.5e2	4.7e2	4.6e2	15	2.6e3	2.1e3	3.1e3	2.6e3

Δf	**f104 in 5-D**, N=15, mFE=22038					**f104 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.7e2	1.3e2	2.2e2	1.7e2	4	6.7e5	4.3e5	1.4e6	1.7e5
1	15	2.9e3	1.9e3	3.8e3	2.9e3	0	*12e+0*	*87e−1*	*14e+0*	1.1e5
1e−1	15	5.2e3	3.7e3	6.7e3	5.2e3	.	.	.	.	.
1e−3	15	7.8e3	5.7e3	9.9e3	7.8e3	.	.	.	.	.
1e−5	15	7.9e3	5.8e3	1.0e4	7.9e3	.	.	.	.	.
1e−8	15	8.7e3	6.6e3	1.1e4	8.7e3	.	.	.	.	.

Δf	**f106 in 5-D**, N=15, mFE=50001					**f106 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.0e2	1.7e2	2.2e2	2.0e2	10	1.7e5	1.2e5	2.6e5	9.1e4
1	15	2.1e3	1.7e3	2.5e3	2.1e3	0	*62e−1*	*29e−1*	*12e+0*	8.9e4
1e−1	15	9.1e3	6.5e3	1.2e4	9.1e3	.	.	.	.	.
1e−3	3	2.3e5	1.4e5	6.8e5	4.7e4	.	.	.	.	.
1e−5	0	*54e−4*	*35e−5*	*18e−3*	3.2e4	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f108 in 5-D**, N=15, mFE=50001					**f108 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.1e3	1.5e3	2.7e3	2.1e3	0	*80e+0*	*63e+0*	*10e+1*	1.1e5
1	7	7.5e4	5.4e4	1.1e5	4.0e4	.	.	.	.	.
1e−1	0	*12e−1*	*49e−2*	*25e−1*	1.4e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f110 in 5-D**, N=15, mFE=50001					**f110 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	14	1.9e4	1.4e4	2.5e4	1.8e4	0	*46e+3*	*28e+3*	*66e+3*	1.3e5
1	1	7.2e5	3.5e5	>7e5	5.0e4	.	.	.	.	.
1e−1	0	*52e−1*	*14e−1*	*85e−1*	2.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f112 in 5-D**, N=15, mFE=50001					**f112 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.5e2	2.0e2	5.1e2	3.5e2	0	*20e+0*	*14e+0*	*23e+0*	1.1e5
1	15	1.1e4	8.2e3	1.5e4	1.1e4	.	.	.	.	.
1e−1	2	3.5e5	1.8e5	>7e5	4.3e4	.	.	.	.	.
1e−3	0	*29e−2*	*94e−3*	*52e−2*	1.8e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f114 in 5-D**, N=15, mFE=50001					**f114 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	5.5e3	4.1e3	7.0e3	5.5e3	0	*34e+1*	*23e+1*	*57e+1*	1.0e5
1	2	3.5e5	1.8e5	>7e5	5.0e4	.	.	.	.	.
1e−1	0	*29e−1*	*73e−2*	*40e−1*	2.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f116 in 5-D**, N=15, mFE=50001					**f116 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	3	2.1e5	1.1e5	6.8e5	2.4e4	0	*16e+3*	*99e+2*	*21e+3*	7.9e4
1	0	*18e+0*	*44e−1*	*57e+0*	3.2e4	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f118 in 5-D**, N=15, mFE=50001					**f118 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.4e3	2.5e3	4.3e3	3.4e3	0	*13e+1*	*88e+0*	*15e+1*	1.4e5
1	13	2.7e4	2.0e4	3.5e4	2.3e4	.	.	.	.	.
1e−1	2	3.6e5	1.8e5	>7e5	5.0e4	.	.	.	.	.
1e−3	0	*32e−2*	*84e−3*	*24e−1*	2.5e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f120 in 5-D**, N=15, mFE=50001					**f120 in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.6e2	2.7e2	6.6e2	4.6e2	0	*20e+0*	*15e+0*	*26e+0*	8.9e4
1	13	2.7e4	2.2e4	3.4e4	2.4e4	.	.	.	.	.
1e−1	0	*45e−2*	*21e−2*	*10e−1*	2.2e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

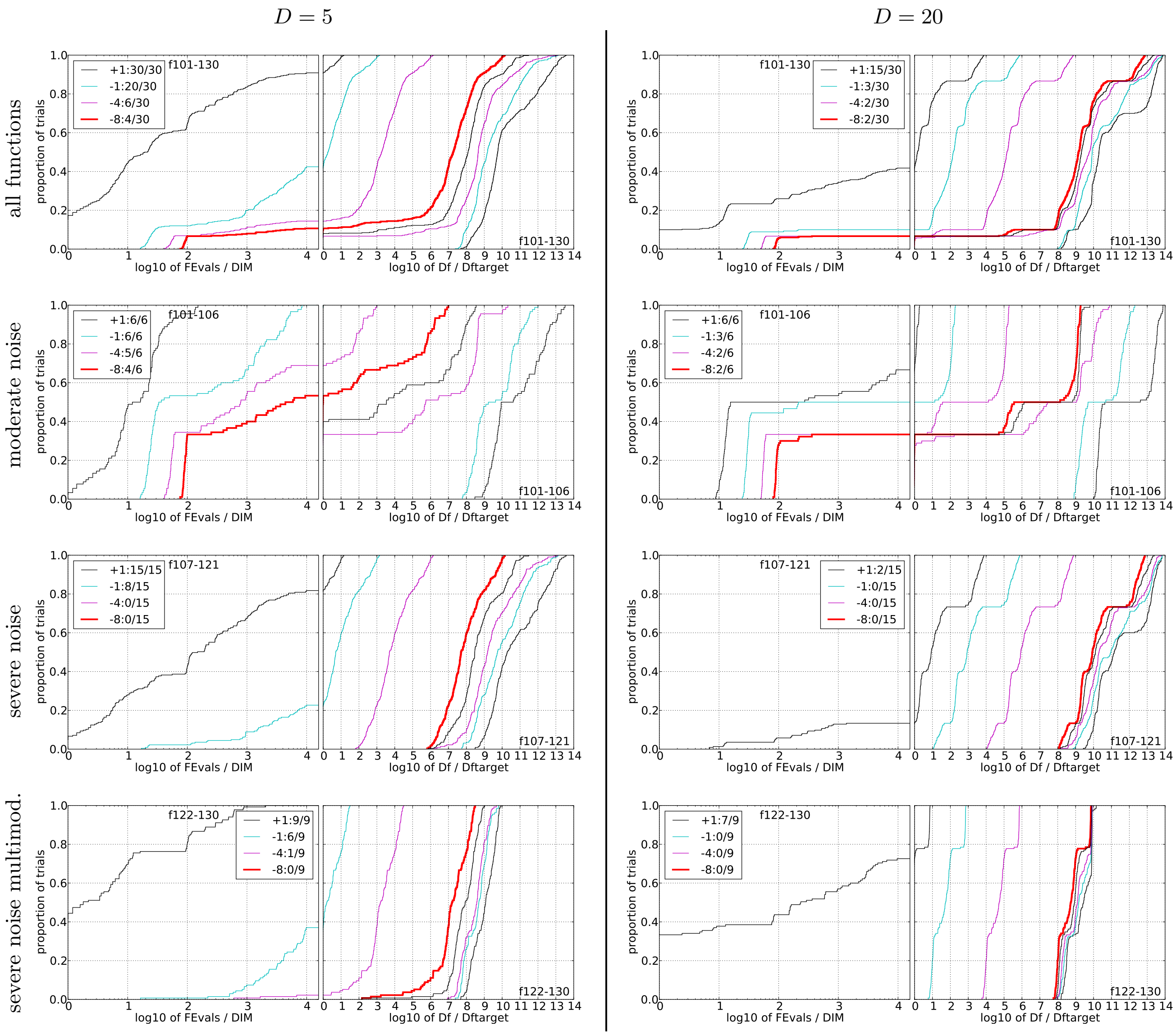

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

Δf	**f_{121} in 5-D**, N=15, mFE=50001					**f_{121} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.4e1	9.7e0	1.8e1	1.4e1	15	8.1e3	5.7e3	1.1e4	8.1e3
1	15	1.1e3	7.0e2	1.5e3	1.1e3	0	*32e−1*	*18e−1*	*43e−1*	1.0e5
1e−1	14	1.6e4	1.2e4	2.1e4	1.6e4	.	.	.	.	.
1e−3	0	*35e−3*	*16e−3*	*66e−3*	2.8e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{122} in 5-D**, N=15, mFE=50001					**f_{122} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.1e2	1.1e2	3.1e2	2.1e2	12	1.0e5	7.5e4	1.3e5	9.2e4
1	6	1.0e5	6.9e4	1.8e5	3.8e4	0	*82e−1*	*72e−1*	*11e+0*	1.0e5
1e−1	0	*12e−1*	*32e−2*	*16e−1*	2.2e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{123} in 5-D**, N=15, mFE=50001					**f_{123} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.0e2	1.4e2	5.0e2	3.0e2	11	1.4e5	1.1e5	1.8e5	1.2e5
1	0	*21e−1*	*12e−1*	*27e−1*	1.6e4	0	*94e−1*	*76e−1*	*12e+0*	7.1e4
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{124} in 5-D**, N=15, mFE=50001					**f_{124} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.6e1	1.3e1	8.1e1	4.6e1	15	2.7e3	1.5e3	3.9e3	2.7e3
1	15	6.6e3	4.9e3	8.4e3	6.6e3	0	*51e−1*	*35e−1*	*66e−1*	7.9e4
1e−1	0	*38e−2*	*24e−2*	*76e−2*	2.8e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{125} in 5-D**, N=15, mFE=50001					**f_{125} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	1.4e2	7.6e1	2.1e2	1.4e2	10	1.9e5	1.5e5	2.6e5	1.3e5
1e−1	11	4.6e4	3.7e4	6.1e4	3.5e4	0	*96e−2*	*72e−2*	*11e−1*	1.0e5
1e−3	0	*75e−3*	*50e−3*	*11e−2*	3.2e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{126} in 5-D**, N=15, mFE=50001					**f_{126} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	4.1e2	2.1e2	6.1e2	4.1e2	2	1.4e6	7.4e5	>3e6	2.0e5
1e−1	4	1.6e5	1.0e5	3.5e5	3.9e4	0	*11e−1*	*97e−2*	*12e−1*	7.9e4
1e−3	0	*12e−2*	*60e−3*	*21e−2*	2.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{127} in 5-D**, N=15, mFE=50001					**f_{127} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e0	1.0e0	1.7e0	1.3e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	5.1e1	1.8e1	8.5e1	5.1e1	15	3.0e4	2.2e4	3.8e4	3.0e4
1e−1	9	5.5e4	3.9e4	7.9e4	3.1e4	0	*81e−2*	*68e−2*	*90e−2*	1.0e5
1e−3	0	*92e−3*	*48e−3*	*13e−2*	2.2e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{128} in 5-D**, N=15, mFE=50001					**f_{128} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.0e3	6.5e2	1.4e3	1.0e3	0	*66e+0*	*56e+0*	*70e+0*	1.1e5
1	13	2.0e4	1.4e4	2.7e4	1.7e4	.	.	.	.	.
1e−1	9	5.3e4	3.8e4	7.7e4	3.3e4	.	.	.	.	.
1e−3	3	2.2e5	1.3e5	6.3e5	5.0e4	.	.	.	.	.
1e−5	2	3.3e5	1.8e5	>7e5	5.0e4	.	.	.	.	.
1e−8	0	*28e−3*	*46e−7*	*10e−1*	2.0e4	.	.	.	.	.

Δf	**f_{129} in 5-D**, N=15, mFE=50001					**f_{129} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.2e3	1.4e3	3.1e3	2.2e3	0	*70e+0*	*66e+0*	*72e+0*	1.0e5
1	8	6.1e4	4.2e4	9.5e4	2.9e4	.	.	.	.	.
1e−1	2	3.5e5	1.9e5	>7e5	5.0e4	.	.	.	.	.
1e−3	0	*77e−2*	*80e−3*	*28e−1*	1.8e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{130} in 5-D**, N=15, mFE=50001					**f_{130} in 20-D**, N=15, mFE=200001				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.5e2	7.5e1	2.2e2	1.5e2	15	1.2e4	8.0e3	1.7e4	1.2e4
1	15	5.5e3	3.3e3	7.8e3	5.5e3	0	*25e−1*	*20e−1*	*50e−1*	1.0e5
1e−1	15	1.4e4	1.0e4	1.8e4	1.4e4	.	.	.	.	.
1e−3	3	2.2e5	1.2e5	6.9e5	3.3e4	.	.	.	.	.
1e−5	0	*71e−4*	*29e−5*	*89e−3*	1.6e4	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 2: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

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